

## IV-6. COUPLED-TRANSMISSION-LINE DIRECTIONAL COUPLERS WITH COUPLED LINES OF UNEQUAL CHARACTERISTIC IMPEDANCES

Edward G. Cristal

Stanford Research Institute, Menlo Park, California

In the past, coupled-transmission-line directional couplers have been designed with coupled lines of equal characteristic impedances.<sup>1-6</sup> These couplers are used in many applications: power samplers, reflectometers, directional detectors, directional filters, and multiplexers are several examples. In this paper a new class of coupled-transmission-line directional couplers, called nonsymmetrical directional couplers, is described. In contrast to conventional directional couplers, nonsymmetrical directional couplers use coupled lines of unequal characteristic impedances. The nomenclature "nonsymmetrical directional coupler" pertains to the side-by-side asymmetry of the directional couplers. It should not be confused with cascaded asymmetrical<sup>4</sup> or cascaded symmetrical<sup>5-6</sup> directional couplers, which use coupled lines of equal characteristic impedances and which have end-to-end asymmetry or symmetry, respectively.

The principal difference between the performance of nonsymmetrical directional couplers and that of conventional directional couplers is the impedance level of the coupled waves, which may be changed to higher or lower impedance levels than that of the incident waves. Nonsymmetrical directional couplers therefore act as conventional directional couplers combined with impedance transformers at the two ports of one of the transmission lines. Thus, nonsymmetrical directional couplers should prove useful in applications calling for directional coupling and impedance transforming in combination, since these functions can be accomplished in a single device.

Figure 1 shows in diagrammatical form the nonsymmetrical coupled-transmission-line directional coupler. The structure in the figure is to be regarded as two uniformly coupled transmission lines of unequal characteristic impedances that have equal propagation constants in balanced (odd-mode) and unbalanced (even-mode) excitation.<sup>1</sup> Define the following:

$$A = \frac{Y_{oo}^a + Y_{oe}^a}{2}, \quad (1)$$

$$B = \frac{Y_{oo}^b + Y_{oe}^b}{2}, \quad (2)$$

$$D = \frac{Y_{oo}^a - Y_{oe}^a}{2} = \frac{Y_{oo}^b - Y_{oe}^b}{2}, \quad (3)$$

where  $Y_{oe}^a$ ,  $Y_{oo}^a$  are the even-and odd-mode admittances<sup>1</sup>, respectively, of line a; and  $Y_{oe}^b$ ,  $Y_{oo}^b$  are the even- and odd-mode admittances<sup>1</sup>, respectively, of line b. Also, let  $G_a$  and  $G_b$  be the terminating admittances of lines a and b, respectively.

By operating on the matrix describing the coupled transmission lines,

it has been shown<sup>7</sup> that the condition for infinite directivity is\*

$$G_a G_b = AB - D^2. \quad (4)$$

There is no other constraint. Note that  $G_a$  and  $G_b$  need not be real to satisfy Eq. (4), although in the remainder of this paper they are so assumed.

By similar mathematical methods, it was also shown<sup>7</sup> that there are two conditions for impedance matching. They are

$$G_a/G_b = A/B, \quad (5)$$

$$G_a G_b = AB - D^2 \quad (6)$$

Note that the condition for infinite directivity is included in the condition for impedance matching. Thus, a matched directional coupler must have infinite directivity. The converse, however, need not be so. A nonsymmetrical directional coupler may have infinite directivity without being matched. (This conclusion may also be established using the scattering matrix for the directional coupler.<sup>8</sup>) Equations (5) and (6) may be solved for  $G_a$  and  $G_b$ , giving

$$G_a = \sqrt{A/B} \sqrt{AB - D^2}, \quad (7)$$

$$G_b = \sqrt{B/A} \sqrt{AB - D^2}. \quad (8)$$

The right sides of Eqs. (7) and (8) are real, positive numbers for all physically realizable values of  $A$ ,  $B$ , and  $D$ . Therefore, the right side of Eqs. (7) and (8) may be interpreted as the characteristic admittances of lines  $a$  and  $b$  in the presence of each other.

Define the coefficient of power coupling as  $k^2$ , where

$$k^2 = D^2/AB. \quad (9)$$

Then, it has been shown<sup>7</sup> that the power relationships for conventional and matched nonsymmetrical coupled-transmission-line directional couplers are identical. The coupling relationships for mismatched nonsymmetrical directional couplers are given elsewhere.<sup>7</sup>

General design equations for nonsymmetrical directional couplers are given below. In these equations choose  $R$  to be the maximum theoretical VSWR (or its reciprocal) desired. (Either of these choices for the value of  $R$  will result in the same power relationships for the directional coupler. However, it will generally be easier to realize the coupler physically for one of the two choices.) In most cases  $R$  will probably be chosen to be 1. However, mismatched nonsymmetrical directional couplers have slightly greater bandwidth than matched nonsymmetrical couplers and may be useful in special applications.

\* Using a low frequency approximation, Firestone<sup>8</sup> showed that the condition for infinite directivity for coupled open-wire lines is (in our notation)  $G_a G_b = C_m / L_m$ , where  $C_m$  and  $L_m$  are the mutual capacitance and inductance per unit length of the coupled lines. It can be shown that this equation and Eq. (4) are equivalent, so that in fact the expression originally derived by Firestone for electrically short couplers is true in general.

DESIGN EQUATIONS FOR NONSYMMETRICAL DIRECTIONAL COUPLERS

$$Y_{oe}^a = \frac{\sqrt{RG_a} - k\sqrt{G_a G_b}}{\sqrt{1 - k^2}} \quad (10)$$

$$Y_{oo}^a = \frac{\sqrt{RG_a} + k\sqrt{G_a G_b}}{\sqrt{1 - k^2}} \quad (11)$$

$$Y_{oe}^b = \frac{G_b/\sqrt{R} - k\sqrt{G_a G_b}}{\sqrt{1 - k^2}} \quad (12)$$

$$Y_{oo}^b = \frac{G_b/\sqrt{R} + k\sqrt{G_a G_b}}{\sqrt{1 - k^2}} \quad (13)$$

The theoretical limit on the amount of impedance transformation and coupling that can be obtained simultaneously can be determined from Eqs. (10) and (12). In order that  $Y_{oe}^a$  and  $Y_{oe}^b$  be positive it is necessary that

$$\frac{1}{k^2} > \frac{RG_a}{G_b}, \frac{G_b}{RG_a} \text{ or } \frac{G_a}{RG_b}, \frac{RG_b}{G_a} \quad . \quad (14)$$

If  $R$  is chosen advantageously, Eq. (14) may be stated in words as follows: Let the impedance (or admittance) transformation ratio of the directional coupler be taken as greater than one. Then, the reciprocal of the power coupling coefficient must be greater than the impedance (or admittance) transformation ratio divided by the maximum VSWR of the coupler.

Matched nonsymmetrical directional couplers may be broadbanded, just as conventional directional couplers, by cascading two or more single-section couplers. Moreover, one may use the existing tables of designs for asymmetrical and symmetrical multisection couplers<sup>4-6</sup> for designing multisection nonsymmetrical directional couplers. The method is to cascade nonsymmetrical directional couplers which, when considered individually, have coupling coefficients corresponding to those given in the tables. Thus, the tables, which are given in terms of even-mode impedances, need only be recast in terms of coupling coefficients. An example is given elsewhere.<sup>7</sup>

EXPERIMENTAL RESULTS

A trial -10-dB matched nonsymmetrical directional coupler having coupled lines of 50 and 75 ohms was constructed and tested. The coupler is shown in Fig. 2 with one of the ground planes removed.

Results of the VSWR measurements are shown in Fig. 3. The measured coupling agreed closely with the theory.<sup>7</sup> The measured directivity was greater than 19 dB.<sup>7</sup>

CONCLUSIONS

Nonsymmetrical directional couplers act as directional couplers with impedance transformers at the two ports of one of the transmission lines. Several possible applications for nonsymmetrical directional couplers are as directional detectors, power samplers, and reflectometers; they may be applied advantageously to the design

of directional filters by permitting a change of impedance level of the resonant lines. A novel application is the construction of impedance transformers having DC isolation either by properly cascading several nonsymmetrical directional couplers to make a 0-dB coupler, or by grounding diagonally opposite ports of a nonsymmetrical -3.01 dB coupler.

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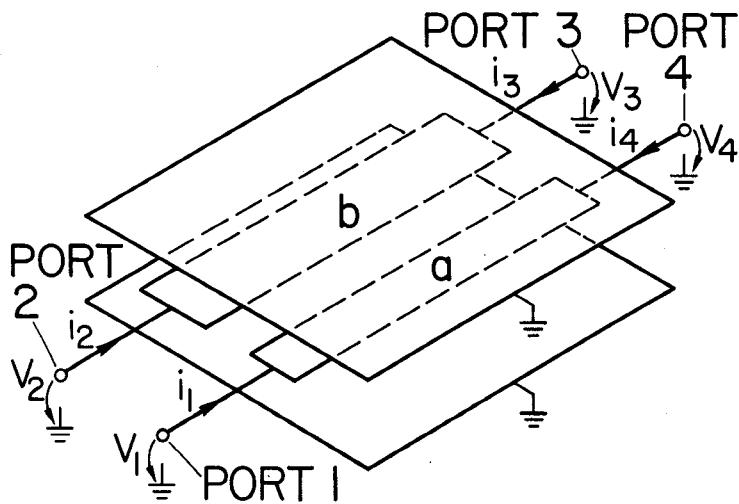


Fig. 1 NONSYMMETRICAL COUPLED-TRANSMISSION-LINE DIRECTIONAL COUPLER WITH COUPLED LINES OF UNEQUAL CHARACTERISTIC IMPEDANCES

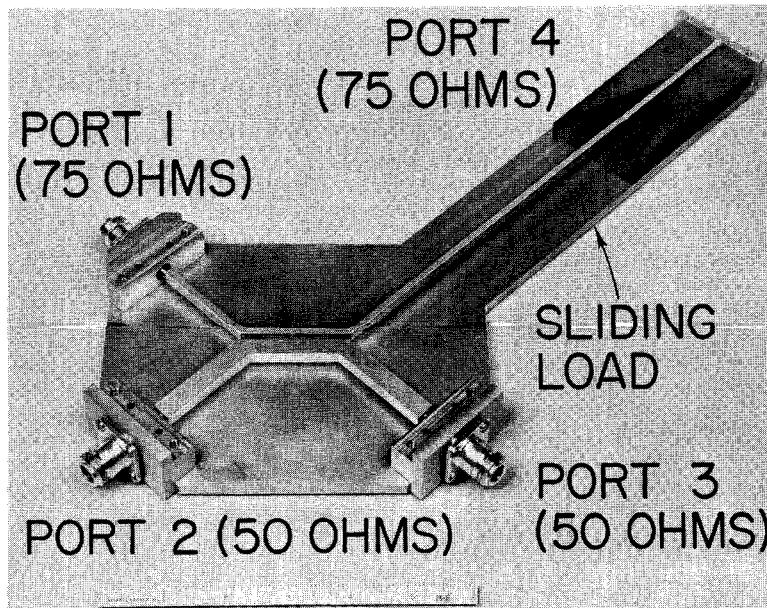


Fig. 2 PHOTOGRAPH OF A TRIAL NONSYMMETRICAL -10-dB DIRECTIONAL COUPLER WITH COUPLED LINES OF 50 AND 75 OHMS

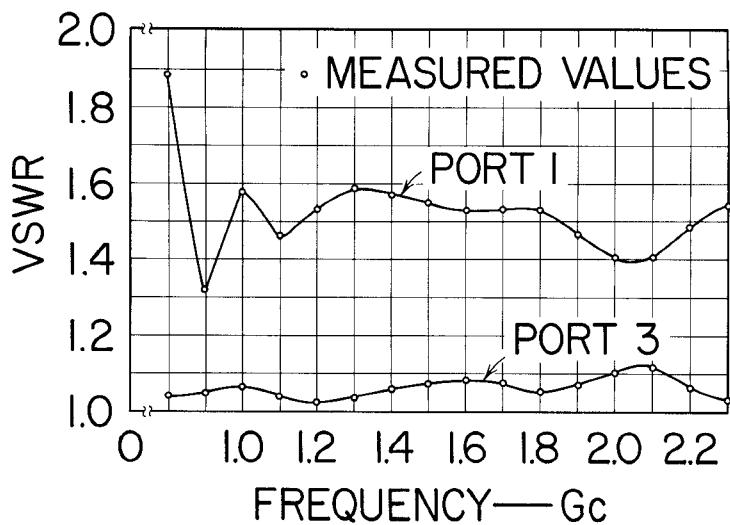


Fig. 3 VSWR AT PORTS 1 AND 3 OF TRIAL NONSYMMETRICAL -10-dB DIRECTIONAL COUPLER. (The VSWR at Port 1 is With Respect to a 50-Ohm Line)